Pension Plan Allocation to Real Estate when Plan Trustees have Reputational Utility

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Abstract

This paper offers a new way to explain the puzzling stylized fact that there is a large mass of institutional investors with very little assets in real estate. The paper develops a model in which pension fund trustees will generally skew their holdings of assets toward investments with the potential for high returns, afraid that if they do not invest in assets with high returns, they may not achieve their target return. To that end, pension fund trustees will devote very little resources to investing directly in real estate. Further, the paper finds evidence that pension fund trustees will conform to group consensus (which explains why there is considerable consensus among institutional investors with respect to their actual real estate allocations). The paper also finds that portfolio allocations are quite persistent over time.

Keywords: Pension Fund; Real Estate; Reputational Utility; Optimism.

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1 Introduction

This paper proposes a different way of thinking about the underinvestment problem in real estate assets by institutional investors. Theoretically, many studies advocate the need for institutional investors to expand their investments in real estate assets (for example, Firstenberg, Ross and Zisler (1988) and Hudson-Wilson (2001)). These studies view real estate as a viable diversifier in an institutional portfolio. Empirically, actual allocations are observed to be only about $2\frac{1}{2}$-4% of total assets, whereas normative amounts range from 15-20% of total assets (Fogler (1984)), to at least 43% (Webb and Rubens (1987)), to a middle ground of 19-28% as suggested by Giliberto (1993).

There are two claims made in this paper. The first is that perhaps a modification of preferences can help us to explain why institutional investors devote very little resources to investing directly in real estate. With this in mind, we assume that pension fund trustees are concerned about how others will assess their performance. Consistent with this contention, we find that (in theory) pension fund trustees will generally skew their holdings of assets toward investments with the potential for high returns, afraid that if they do not invest in assets with high returns, they may not achieve their target return. With the target return set high enough, this fear of failure forces pension fund trustees, among other things, to be optimistic in their expectations. Further, because it is better for reputation to fail conventionally, there is pressure for pension fund trustees to conform to group consensus (which explains why there is considerable consensus among institutional investors with respect to their actual real estate allocations). In addition, what clearly emerges here is a finding that portfolio allocations are quite persistent over time.

The second claim is that with an excessively high allocation to investments with the potential for high returns, there is an accompanying decrease in the allocation to real estate. Theoretically, real estate provides variance reduction benefits to pension funds, but the variance reduction benefits are offset by lower expected returns and, as such, necessarily detracts from the pension fund trustee's ability to meet the target rate of return on the portfolio. For this reason, most institutional investors have a low allocation to real estate.

We test these ideas by looking at stylistic differences between pension fund managers, which we estimate following Sharpe (1992). We then take the percentage of the fund's target return that is explained by movements in the benchmark returns and regress this on the fund's target surplus return (the difference between the fund's long-term return on plan assets and its average realized return on liabilities) using a logistic transformation of the data and a logistic regression. As the fund's target surplus return increases, we generally expect the pension manager's target return to be more explained by movements in the benchmark returns (i.e., pension fund managers should become more conformists to a particular style or multiple styles). We also test to see whether pension fund conformity is a function of the initial funding ratio. Our findings suggest that when the initial funding ratio is high, pension plans rely less on asset allocation but more on security selection at an increasing rate to achieve a given target surplus return.

Lastly, we simulate the asset allocation model to show that the optimal pension fund portfolio is concentrated in stocks and bonds, with relatively little allocated to real estate. The latter generally ranges from few percent up to 8%. These results are by and large consistent with what is applied in practice.

The remainder of the paper is organized as follows. Section 2 contains a formal model description and outlines our computational techniques. Section 3 presents an empirical test of the model in which we consider whether high target returns cause pension fund managers to become conformists. Section 4 presents some simulation evidence bearing on the practically of the theoretical model in explaining the underinvestment problem in real estate assets by institutional investors. Section 5 concludes.

2 The Model

Our objective is to explain pension portfolio choice when reputation is an argument of the pension fund trustee's utility function. We measure the reputation of the kth pension fund trustee using the consequences drawn from the entropy function H:

$$\max_{\boldsymbol{\alpha}} \max_{q(x^k(\boldsymbol{\alpha}))} H\left(q(x), ES(\boldsymbol{\alpha}, q); x^k(\boldsymbol{\alpha})\right) \tag{1}$$

where $\boldsymbol{\alpha}' = (\alpha_1, \alpha_2, \ldots, \alpha_N)$ represents the asset weights of N assets in the portfolio and is subjected to the usual constraints of $\alpha_i \geq 0, \sum \alpha_i = 1$. q is an epistemic probability of the mean portfolio return x^k. That is, q may or may not coincide with the true probability of x^k occurring, depending on the kth trustee's beliefs as to what rate of return is achievable, and on the average opinions of other pension fund trustees. x takes on the values of the $\left\{x^1, x^2, \ldots, x^K\right\}$ with respective probabilities $\left\{q^1, q^2, \ldots, q^K\right\}$. Here, H describes the amount of ordering and disordering that occurs for each choice of q^k (equivalently, $q\left(x^k\right)$) and $\boldsymbol{\alpha}$. Furthermore, we assume that there is a single pension fund trustee who determines x^k and $\boldsymbol{\alpha}$ for each pension plan in the community of K trustees. The justification for using the entropic specification in (1) derives from our presumption that pension fund trustees not only care about their professional reputation, but will avoid those selections of q^k and $\boldsymbol{\alpha}$ that will lead to large losses and a loss of reputation.

For each trustee[1], the maximization problem in (1) is subject to a constraint that limits the amount of downside risk ES in the portfolio. The variable ES is defined as the expectation of a shortfall:

$$-ES = \int_x \int_g (x - g)\, F_x(g)\, h(g)\, q(x) \mathrm{d}g \mathrm{d}x \tag{2}$$

where x and g are random variables whose realizations are draws from their respective probability distributions $q(\cdot)$ and $h(\cdot)$. Both x and g are assumed independent.

Equation (1) can be understood as two related problems: (a) given a set of portfolio decisions, what is a trustee's belief choice? and (b) with an

[1]For ease of exposition, we drop the superscript k and include it when such inclusion makes the discussion henceforth clearer.

associated optimal belief choice for each portfolio, what is the optimal portfolio allocation? We look at each of these problems in turn in the following sections.

2.1 Trustee Behavior and Optimal Belief Choice

In this section, we want to solve the inner maximization of (1) for an optimal $q(x^k)$ over a given set of $\boldsymbol{\alpha}$ and a given ES,

$$\max_{q(x^k(\boldsymbol{\alpha}))} H\left(q(x), ES(\boldsymbol{\alpha}, q); x^k(\boldsymbol{\alpha})\right)$$

This translates to a more convenient form

$$\begin{aligned} &\max_{q(x)} \quad H = \max_{q(x)} - \int q\left(x\right) \log q\left(x\right) \mathrm{d}x \\ &\text{s.t.} \\ &\qquad \int q\left(x\right) \mathrm{d}x = 1 \\ &\qquad \int_x \int_g \left(x-g\right) F_x\left(g\right) h\left(g\right) q(x) \mathrm{d}g \mathrm{d}x = -ES \end{aligned}$$

where $-\log q(x)$ represents the amount of surprise evoked if the mean portfolio return takes on the value x. In the case when $q = 0$, we take $0 \log 0$ be 0.

For tractability, assume that x takes on discrete values. The optimal belief choice, obtained via calculus-of-variation is

$$q^*\left(x\right) = \frac{e^{\theta(c+d[x-\mu_g])}}{\sum_{k=1}^{K} e^{\theta(c+d[x-\mu_g])}} \tag{3}$$

Here, θ is the usual variational calculus multiple, μ_g is the mean target return across all pension plans, $c \equiv -\sigma_g^2$ is the (negative of) cross-sectional variance of g across all pension plans, and $d \equiv (\mu_g - \underline{x})$, where $\underline{x}$ is the lower bound of the support of x.

Expression (3) is a multinomial logit probabilistic choice function. In this model, at a given level of θ, q^* is higher if trustees are inherently more optimistic (to see this, insert $x > \mu_g$ into (3)). The intuition behind this result is quite simple. Trustees will generally skew their holdings of assets toward investments with a high return (i.e., $x > \mu_g$), fearing that if they do not, they may not achieve their target return.

Further, there is a tendency to conform to the average μ_g when everyone else chooses that way. To see this, suppose that the kth plan trustee chooses $x^k = \mu_g$. Let us further suppose that all remaining plan trustees choose varying x's that are higher than μ_g. Then it follows from (3) that

$$q\left(x^k = \mu_g\right) = \frac{1}{1 + \sum_{x^h \neq x^k,\, x^h > \mu_g} \exp\left(\theta\left(d\left[x^h - \mu_g\right]\right)\right)}$$

Here the term $\sum_{x^h \neq x^k,\, x^h > \mu_g} \exp\left(\theta\left(d\left[x^h - \mu_g\right]\right)\right)$ is increasing in x^h. Hence, the probability of choosing a value of x^k equal to μ_g declines as x^h increases. Similarly, when everyone else selects a x^h less than μ_g, the probability of choosing a x^k equal to μ_g is quite small. This result suggests that pension plan trustees are likely to be conformists and therefore are likely to have the same x^k. Hence, we obtain Keynes' result that it is better for reputation to fail conventionally.

Next, we turn to the choice of x as the volatility and disagreement in x shrinks. We explore this in a binary discrete choice case. Let us suppose that there are only two choices for the values of x. One choice is to select μ_g. The other choice is to select $x \neq \mu_g$. We can represent the probability of the latter as

$$q(x) = \frac{\exp\left(\theta \cdot d\left(x - \mu_g\right)\right)}{\exp\left(\theta \cdot d\left[x - \mu_g\right]\right) + 1}, x \neq \mu_g \tag{4}$$

From (4), $x > \mu_g$ is the most likely choice, since q increases with the magnitude of $(x - \mu_g)$. Hence, when there is a limited number of choices, pension fund managers will generally prefer the more optimistic choice (i.e., a higher x), because of the need to achieve a certain target rate of return on the portfolio.

The multinomial-logit form of (3) enables us to use ideas from discrete choice theory to explicate why pension plans can have persistent beliefs. The exposition here follows the work of Brock and Durlauf (1999). We limit our attention to consider only two choices by trustee k: x^1 and x^2. Without loss of generality, denote the choice of x^1 as 1 and the choice of x^2 as -1. Then

$$Pr\{\text{trustee } k \text{ chooses } +1\} = \frac{\exp\{\theta\,(\mu_g - \underline{x})\,(x^2 - x^1)\}}{\exp\{\theta\,(\mu_g - \underline{x})\,(x^2 - x^1) + 1\}} \tag{5}$$

Assume that all trustees expect the mean μ_g to be (a) the same as it was last period and (b) to be determined by the average trustees' choice of belief, then μ_g at time t can be related to the trustees' choices via a phase diagram given by the equation

$$\begin{aligned} \mu_g(t) &= \Pr\{\text{choice of } +1\} - \Pr\{\text{choice of -1}\} \\ &= \frac{exp\{\theta[\mu_g(t-1) - \underline{x}]\left(x^2 - x^1\right)\} - 1}{exp\{\theta[\mu_g(t-1) - \underline{x}](x^2 - x^1)\} + 1} \end{aligned} \tag{6}$$

Equation (6) is a deterministic function and is dependent on the magnitudes of three parameters: that of θ, the value of $\underline{x}$ (which will be the lower of x^1 and x^2), and the size of $(x^2 - x^1)$. The graph of μ_g is depicted in Figure 1 and shows how the shape of μ_g changes as the parameters change. Specifically, it illustrates the idea that the target return based on the above conditions can be rather persistent.

Figure 1 depicts the first-order dynamic relationship when trustees perceive the mean target return today to be the same as what has transpired last period. The 45 degree line indicates the alternative stable states. The function $\mu_g(t)$ increases with $\mu_g(t-1)$.

Further, when θ is zero, the choice of $\mu_g(t)$ will also be 0, suggesting that trustees randomize their choices: there is a 50-50 chance of which prediction they will choose, resulting in half of them choosing either belief index. We

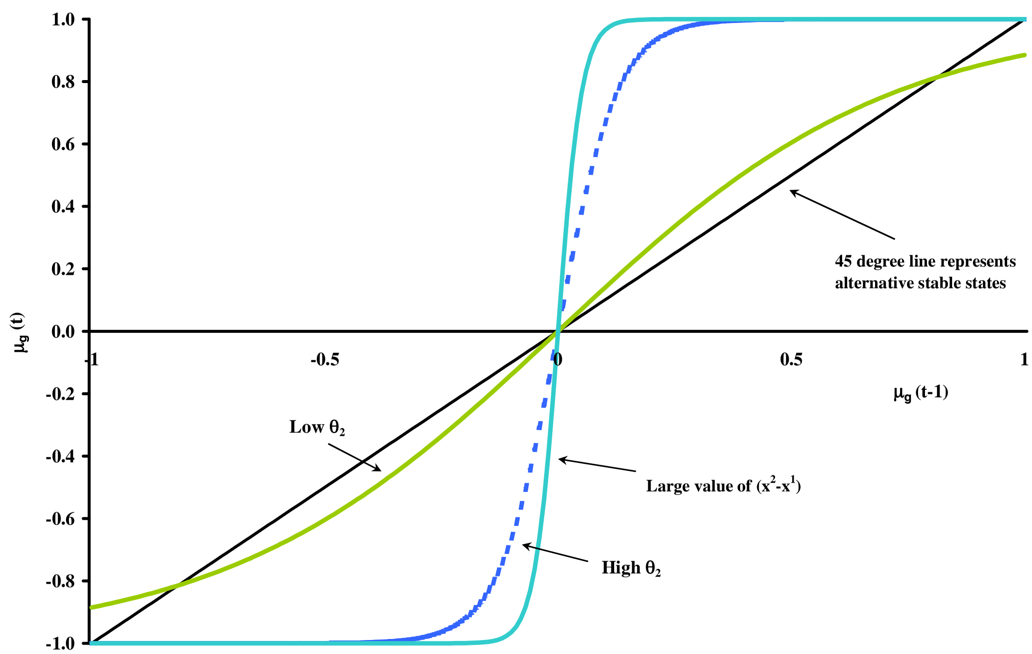

Figure 1: Persistence in beliefs

also see that an increase in θ and $(x^2 - x^1)$ pivots the graph at 0 and twists it so that there is a steep increase from -1 to +1 as $\mu_g(t-1)$ increases from negative to positive. Notice that this creates a sort of sluggishness on the movement of $\mu_g(t)$. To see this recall that as $\mu_g(t-1)$ moves from -1 toward +1, high values of θ or $(x^2 - x^1)$ will "trap" this past choice of -1: $\mu_g(t)$ will stay adamantly at -1 and the equilibrium value will be at -1 unless $\mu_g(t-1)$ moves beyond a critical value past 0.

On the other hand, keeping $(x^2 - x^1)$ constant while decreasing the value of θ gives a softly-sloped graph. In this situation, the stable states are less extreme: the mean target return is perceived to be less biased toward -1 and is more responsive to changes in $\mu_g(t-1)$ but the stable states remain at 3 points. The gently sloping graph of low values of θ is an interesting case because any shift in the curve, upward or downward, will only result in one stable state. This is shown in Figure 2.

Figure 2 shows different stable states under a same unit increase or decrease of the lower bound of x. When $\underline{x}$ increases, it shifts the function $\mu_g(t)$

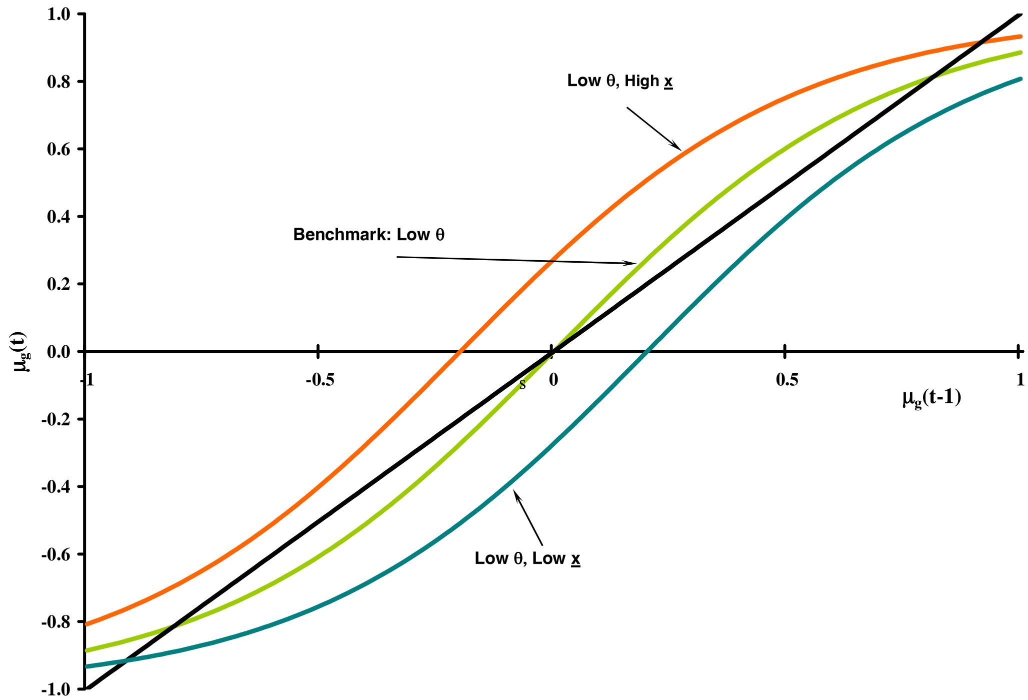

Figure 2: Varying lower bound $\underline{x}$

upward; and when $\underline{x}$ decreases, it shifts the function downward[2]. One can think of a time-varying $\underline{x}$ as indicating the state of *optimism*. When the value of the lower bound of mean return is extremely low, the mean target return among trustees will be set low, indicating pessimism. On the other hand, when the lower bound *underlinex* increases by a dramatic amount, the mean target return among trustees will be raised.

Notice that one can independently derive (3) in a random utility setting whereby θ has a *intensitiy of choice* interpretation. However, it has a more relevant interpretation in the portfolio theory. We show in the next section that θ is directly related to a trustee's relative risk aversion. This is obtained from the simple parallelism between an entropy-maximizing analysis and a power utility-maximization problem via the *Legendre-Fenchel transform*.

[2]Note that $\underline{x}$ is set exogenously.

2.2 The Optimal Allocation of the Portfolio

Having solved the inner maximization problem in (1) we now have to determine the optimal allocation:

$$\max_{\boldsymbol{\alpha}} H\left(q^*(x), ES(\boldsymbol{\alpha}, q^*); x^k(\boldsymbol{\alpha})\right)$$

It turns out that by assuming some regularity conditions, one could actually interpret the entropy-maximization analysis above as maximizing the familiar *power utility* function via a useful mathematical representation: the *Legendre-Fenchel transform*. To see this, define the *cumulative generating function* (cgf) $c_\rho(\theta)$ of a portfolio surplus return as a function of a real-valued parameter $\theta > 0$,

$$c_\rho(\theta) \quad = \quad \log E_\rho\left[\left(\tfrac{S_T}{W_0}\right)^\theta\right] \tag{7}$$

where W_0 is the beginning dollar value of plan assets and surplus return S_T is the difference between a plan's assets and its liabilities at the end of the holding period T, i.e. $S_T = W_0 \prod_{t=1}^{T} (\boldsymbol{\alpha}'(\mathbf{1} + \boldsymbol{r}_t) - l_t)$. The parameter $\boldsymbol{r}$ represents a vector of N risky asset returns where $1 + r_{i,t} \geq 0, i = 1, 2, \ldots, N$

$$\boldsymbol{r}' = (r_{1,t}, r_{2,t}, \ldots, r_{N,t}),$$

and l_t is the pension liability growth rate between period $t-1$ and t. By assuming that $c_\rho(\theta) < \infty$, Theorem II.4.1 of Ellis (1985) shows that, for any real-valued z, there exists a *decay rate* $I_\rho(z)$ which is the rate at which the probability of a shortfall goes to zero as the holding period increases, i.e. $T \rightarrow \infty$:

$$I_\rho(z) = \max_{\theta \in \Re} \{\theta z - c_\rho(\theta)\} \tag{8}$$

Equation (8) is, in fact, the *Legendre-Fenchel* transform of $c_\rho(\theta)$. Fortuitously, it turns out that by a *contraction principle*, I_ρ is the maximum of an entropy of a distribution, i.e. $I_\rho(z) = H^{max(q)} = H(q^*(x))$ except that now, we have I_ρ as a function of the variable z while $H(\cdot)$ is actually a functional of the distribution q. The Legendre-Fenchel transform, given the positive parameter θ, provides a convenient way for us to associate the choice of a plan trustee's belief with a familiar utility functional form. It can be shown that the asset allocation problem in (1) can be written as:

$$\max_{\boldsymbol{\alpha}} H(q^*(S_T), \log z) \equiv \max_{\boldsymbol{\alpha}} \max_{\theta} \{\theta \log z - c_\rho(\theta)\} \tag{9a}$$

$$= \max_{\boldsymbol{\alpha}} \max_{\theta} \left\{ \log z^\theta - \log E_\rho \left[\left(\frac{S_T}{W_0} \right)^\theta \right] \right\} \tag{9b}$$

$$= \max_{\boldsymbol{\alpha}} \max_{\theta} \left\{ \log E_\rho \left(\frac{S_T}{W_0 z} \right)^{-\theta} \right\} \tag{9c}$$

The first identity replaces z with $\log z$ without lost of generality. We then substitute the definition of $c_\rho(\theta)$ from (7) and rearrange the terms to arrive at the last equality. $W_0 z$ represents the (dollar value of the) targeted portfolio surplus. Exponentiating (9), we see that

$$-e^{\max_{\boldsymbol{\alpha}} H(q^*(s_T), \log z)} = \max_{\boldsymbol{\alpha}} E_\rho \left[- \left(\frac{S_T}{W_0 z} \right)^{-\theta_{\max}(\boldsymbol{\alpha}, \log z)} \right] \tag{10}$$

Equation (10) is similar to a conventional expected power utility form of $EU = E\left[-(W_T)^{-\theta}\right]$ where U represents the utility of an agent and $\gamma = 1 + \theta_{max}(\boldsymbol{\alpha})$ represents the Arrow-Pratt relative risk aversion parameter. The trustee's strictly prefer a plan surplus that is higher than a target surplus denoted by $W_0 z$ (monotonicity) and he is strictly averse to risk (concavity). The above properties arise directly from $U'(\cdot) > 0$ and $U''(\cdot) < 0$. Because $\theta > 0$, the third derivative is positive. This implies that the trustee prefers positively skewed surplus returns. This corroborates research by Arrow (1963), Pratt (1964), Kraus and Litzenberger (1976) and more recently Harvey and Siddique (2000).

In our discussion above, we treated z as the target surplus return. This is non-trivial and results from the following proposition:

Proposition Given that $H^{max(q)} = \max_{\theta\in\Re} \{\theta z - c_\rho(\theta)\}$, z is the *perceived* surplus of a plan trustee: z is the expected surplus according to his chosen belief $q^*(x)$.

Proof: Start with (8), a maximum is obtainable *if and only if*

$$\frac{\partial}{\partial\theta}\{\theta z - c_\rho(\theta)\} = 0$$

which implies

$$c'_\rho(\theta) = z$$

Note that

$$\frac{\partial}{\partial\theta}\int_\Re \exp(\theta s)\,\rho(ds) = \int_\Re y\exp(\theta s)\,\rho(ds)$$

where $\rho(ds)$ is the *probability distribution function* (pdf) of surplus return s[3]. Then

$$c'_\rho(\theta) = z = \int_\Re s\frac{\exp\{\theta s\}}{\int_\Re \exp\{\theta s\}\cdot\rho(ds)}\rho(ds)$$

One can alternatively express ρ in terms of its Radon-Nikodym derivative ρ_θ such that

$$\frac{d\rho_\theta}{d\rho}(s) = \exp\{\theta s\}\cdot\frac{1}{\int_\Re \exp\{\theta s\}\,\rho(ds)}$$

Note that ρ_θ represents the "equivalent" probability measure of the true distribution of the surplus return s. In the present context, the

[3]Note that the dollar value of a plan's surplus is S_T whereas its surplus return is denoted as a lower-case s.

choice belief $q(x) \equiv \rho_\theta$. Let $s = s_T$, then z is the perceived plan's surplus.

$$c'_\rho(\theta) \equiv \sum_{\Re} s_T \frac{\exp\{\theta s_T\}}{\sum_{\Re} \exp\{\theta k\}} = z \tag{11}$$

Q.E.D.

The proposition relies on the use of the Radon-Nikodym derivative ρ_θ of the true distribution of the portfolio surplus. This results in an equivalent measure that shifts the mean of the surplus distribution but retains the variance structure. One should note that such an *equivalency* or a *mean-shifting* occurs in the mind of the plan trustee since this same probability belief $q^*(x)$ is the belief that the trustee has chosen to maximize his decision distribution entropy.

The novelty of this proposition lies in the internal consistency of a *distortionary action*. The plan trustee "self-imposed" a target portfolio surplus return z. This z is formed via his ex-ante belief of the distribution of x^k. The ex-ante belief is an equivalent measure; it is distorted to the extent that the expected surplus return based on it (i.e. z) is not the same expected plan surplus based on the true return-liability distribution. Nonetheless, this belief has the same variance structure as the true return-liability series. This is consistent with the current literature on variance estimation: Merton and many others have noted that expectations are prone to distortion whereas variance has been proven to be less problematic in its estimation.

2.3 Implications

2.3.1 Initial Funding Ratio is important

The argument of the expected utility in (10) has a *beating-the-benchmark* flavor where z is the benchmarked portfolio surplus. In particular, for a particular plan, we can express the ratio of its realized surplus to its targeted surplus in terms of a plan's funding ratio:

$$\frac{S_T}{W_0 z} = \frac{W_T(\boldsymbol{\alpha}) - L_T}{W_B - L_B}$$

where $W_T(\boldsymbol{\alpha})$ and L_T are the time-T dollar values of a plan's asset and liability respectively, W_B is the benchmark asset value W_B and L_B is the benchmark liability value. Dividing the numerator and the denominator of the RHS by the initial liability value L_0, we have

$$\frac{W_T(\boldsymbol{\alpha}) - L_T}{W_B - L_B} \div \frac{L_0}{L_0} = \frac{\frac{W_T(\boldsymbol{\alpha})}{L_0} - \frac{L_T}{L_0}}{\frac{W_B}{L_0} - \frac{L_B}{L_0}} = \frac{IF_0 \times \boldsymbol{\alpha}'(\mathbf{1} + \boldsymbol{r}_t) - l_T}{IF_0 \times (1 + r_B) - l_B} \tag{12}$$

IF_0 represents the initial funding ratio of the plan, $1 + r_B$ is the benchmark asset gross return, l_T represents the actual liability net return at period T and l_B represents the benchmark liability net return. In contrast to Leibowitz et al. (1995), the results here indicate that initial funding ratio scales a plan trustee's utility function. Consequently, there is "no universal" measure in developing a one-for-all allocation strategy for all pension plans of the same risk tolerance. Rather, asset allocation differs among pension plans not only because they each have differing exposures to liability or assets' movements, but also because their initial funding ratios are different.

2.3.2 Risk aversion is related to the Opportunity Set

The present model implies a risk aversion parameter, γ, that is equal to $1 + \theta_{\max}(\boldsymbol{\alpha}, \log z)$. The presence of the inner maximization over θ prior to maximizing over different portfolios $\boldsymbol{\alpha}$ means that a trustee would have to jointly evaluate the opportunity set he faces as well as how optimally risk tolerant he would want to be. This stands is contrast to Sharpe's Capital Market Pricing Model (CAPM) and Markowitz's portfolio theory where the trustee's risk aversion parameter is exogenous. In this model, an investor's risk aversion is dependent on the feasible portfolios $\boldsymbol{\alpha}$'s (Stutzer, 2003).

2.3.3 Risk aversion is related to Optimism and Confidence

While $\theta_{max}(\boldsymbol{\alpha})$ is directly related to the risk aversion parameter, it can also be interpreted as the *intensity of choice* in a probabilistic-choice setting. Equation (3) can be separately derived from a discrete choice model where trustees have a utility function $\tilde{U}$ that is inherently stochastic: specifically, a trustee's utility function has a deterministic component $U\left(x^k, \mu_g\right)$ and a stochastic component ϵ_k. The deterministic component is simply $(x^k - \mu_g)$ and the stochastic component[4] is a random variable that is mutually and serially independent extreme value distributed:

$$\tilde{U^k} = U\left(x^k, \mu_g\right) + \frac{\epsilon_k}{\theta}$$

$\theta_{max}(\boldsymbol{\alpha})$ in (10) is equivalent to the product (θd) in (4). Consider (4), θ is the *intensity of trustee's choice* of prediction x in the sense that as θ increases from zero to infinity, $q\left(x\right)$ goes from a horizontal line at $\frac{1}{2}$ to a function that is zero for $x < \mu_g$, $\frac{1}{2}$ at $x = \mu_g$, to 1 for $x > \mu_g$. That is, as the intensity of choice increases from zero to infinity the probability that a trustee chooses a particular x that is higher than μ_g goes to unity. In other words, as risk aversion increases (increasing θ), the probability of a trustee being more *optimistic* about his portfolio's performance (a high x) increases dramatically.

As is well-known in discrete choice models, θ also scales the amount of uncertainty in the choice of a particular predictor x^k and varies inversely with the standard deviation of ϵ_k, σ_ϵ^k (Anderson, de Palma and Thisse, 1992). When a trustee is confident of his reputation utility, there is less uncertainty in this utility function and θ will be high (as $\theta \to \infty$, $\frac{\epsilon_k}{\theta} \to 0$). On the other hand, when a trustee sees his reputation utility function as being less certain, θ will be low and consequently the trustee would have a more uncertain utility

[4]There are generally two reasons to account for the randomness of the individual's utility function: either the econometrician modelling the plan trustee's choice of action faces unobservables or that these trustees have utility functions that are dependent on the realization of random variables. Since the trustee themselves form a multinomial logit function of their choice, the latter assumption that these individuals have stochastic utilities is more apt in this set-up.

function. In summary,

Table 1: Trustees' Risk aversion, Optimism and Confidence

θ	High	Low
Risk Attitude	Risk-averse	Risk-loving
Optimism	Increases	Decreases
Confidence	Increases	Decreases

Against the duality that θ also represents relative risk aversion, the model says that a trustee is more confident when he is more certain of his environment. Curiously, this confidence does not translate to being more risk-loving - rather, a confident trustee is actually more risk averse (having a low θ). Conversely, a trustee who is more uncertain about his environment would randomize his choice of x^k and is therefore more risk-tolerant since he needed that particular risk-loving behavior to deal with the uncertainty he perceives.

2.3.4 Risk-averse Conformists as Herds

Expression (3) nests the special case whereby all trustees have homogeneous expectations (i.e., $q(x^k) = 1$.) This happens when there is only one belief choice (no diversity) or when θ is very large ($\theta \to \infty$) High risk aversion therefore implies that a trustee perceives less diversity in terms of expected returns on portfolios across different plans. A trustee is therefore more likely to conform. The model therefore explains herding behavior as a case where the all plan trustees are conformists and have rather high risk aversion. As we've discussed in the earlier section, high risk aversion is associated with increased optimism and confidence and these sentiments in turn lead to more trend-chasing activity such as funds indexing.

In addition, recall that high values of θ, in the case of binary beliefs results in μ_g being more persistent, thus implying that one would expect more inertia in a more risk-averse (certain) environment than in the case of a more risk-loving (uncertain) environment. While the model does not point to the direction of causality[5], it suggests that a market of persistent beliefs is associated with more-risk averse investors.

2.3.5 Reducing the risk of ruin and relation to the Sharpe Ratio

Not unlike the *large deviation property*, equation (9) implies that a trustee chooses an optimal portfolio that has a sample surplus return which converges quickly to the true mean surplus return. While a trustee is allowed an opinion on the distribution of his surplus return that may not be the same as the true probability distribution ρ, however, as the number of observed sample mean increases, his perceived distribution should approach the true mean exponentially. This rate of convergence differs with different portfolios; our model says that a reputation-caring trustee would choose a portfolio that has the highest rate of convergence. This means that in choosing the portfolio with the optimal rate of convergence, a trustee chooses a belief that is relatively close to the true underlying probability distribution of the surplus returns. This also means that while a trustee has the freedom to choose his beliefs, he will not choose an *incredulous* belief which leads to an ultimate disaster. Put differently in probabilistic terms, he will choose a belief that reduces the *risk of ruin.* A trustee's reputation therefore depends on how well he perceives his investment environment as well as how well he can avoid the risk of ruin.

Interestingly, the idea of avoiding ruin has a Markowitz mean-variance counterpart to it. Consider the Bienaymé-Tchebycheff[6] inequality concerning the portfolio surplus return $\tilde{s}_T \sim (\mu_s, \sigma_s)$,

$$P\left(\mid \tilde{s}_T - \mu_s \mid \geq \mu_s - d\right) \leq \frac{\sigma_s^2}{\left(\mu_s - d\right)^2}$$

Then *a fortiori*,

$$P(\mu_s - \tilde{s}_T \geq \mu_s - d) = P(\tilde{s}_T \leq d) \leq \frac{\sigma_s^2}{\left(\mu_s - d\right)^2}$$

[5] More risk-loving attitude causes more uncertainty and vice versa.

[6] This section abstracts from a discussion in Roy (1952).

In reducing the probability that the final portfolio surplus return will be less than a *floor return* of d, we are in fact maximizing $(\mu_s - d)/\sigma_s$. Replacing this floor return d with the risk-free rate, r_f, we get the Sharpe Ratio $(\mu_s - r_f)/\sigma_s$. The portfolio with the maximum Sharpe Ratio is also the one which minimizes the probability of its surplus return dropping below the rate of return on treasury bills. In the CAPM world, this is the tangency portfolio. The main difference between the model in this paper and the CAPM is that we have made explicit the choice of a risk-aversion parameter and a belief choice.

3 Empirical Evidence of Conformity Effect

In this section, we test the implications discussed above. In particular, we provide evidence that a high target surplus return indeed causes pension fund managers to become more conforming to a particular investment style or multiple styles. Furthermore, we show that conformity effects are higher when (a) everyone else chooses that way and (b) volatility increases.

3.1 Measuring Conformity to Investment Styles

We use Sharpe's (1992) Asset Class Factor Model (ACFM) to determine the investment style of each pension plan. In Sharpe's model, returns on style benchmark index portfolios are used to replicate the return on a managed portfolio as closely as possible. Equation (13) gives a generic n-factor model that decomposes the return on a managed portfolio i into different components:

$$\tilde{A}_{it} = b_{i1}\tilde{F}_{1t} + b_{i2}\tilde{F}_{2t} + \ldots + b_{in}\tilde{F}_{nt} + \tilde{e}_{it} \tag{13}$$

$\tilde{A}_{it}$ represents the managed pension fund portfolio's expected return at time t, $\tilde{F}_{1t}$ represents the return on the style benchmark portfolio 1 at time t, $\tilde{F}_{2t}$ represents the return on the style benchmark portfolio 2 at time t, $\tilde{F}_{nt}$ represents the return on the style benchmark portfolio n at time t, and $\tilde{e}_{it}$ is the non-factor component of the return. The coefficients $b_{i1}, b_{i2}, \ldots, b_{in}$

represent the managed portfolio average allocation among the different style benchmark index portfolios during the relevant time period. These portfolio weights must sum to unity.

Data for $\tilde{A}_{it}$ are for Compustat firms for the 15-year period from 1990 to 2004. A total of 421 firms provide information on their pension plan's funding status, the company's expected return on pension fund assets, and the pension fund's projected benefit obligation (PBO). The Financial Accounting Standards Board (FASB) requires these accounting and disclosures.

We use 6 asset classes for $\tilde{F}_{1t}, \tilde{F}_{2t}, \ldots, \tilde{F}_{6t}$. These include Bills (Cash equivalent with less than 3-months to maturity), long term government bonds (Government bonds 10 years and over), corporate bonds, the Small-cap equity, Large-cap equity and Real Estate. Except for the real estate returns series which is the total return REIT series from the NAREIT website, the rest of the data are obtained from the Ibbotson Associates' SBBI 2005 Yearbook.

A standard variance decomposition is used to decompose pension plan investment behavior into style and non-style effects. Pension plans that depart from a benchmark style will have a lower explanatory power and the residual terms $\tilde{e}_{it}$ will be large. In contrast, pension plans that conform to a particular style or multiple styles will have a higher explanatory power and the residual terms will be low. As a result, this suggest that

$$R_i^2 = 1 - \frac{\mathrm{Var}(\tilde{\mathrm{e}}_{\mathrm{it}})}{\mathrm{Var}(\tilde{\mathrm{R}}_{\mathrm{it}})} \tag{14}$$

can be used to determine the proportion of the variance of $\tilde{R}_{it}$ that is explained by the n factors.

The distribution of R_i^2 has the undesirable characteristic of being bounded between 0 and 1, and cannot be, strictly speaking, normally distributed. To circumvent this problem, we apply a logistic transformation to R_i^2. More specifically, we measure

$$\Psi_i = \log\left(\frac{\mathrm{R}_{\mathrm{i}}^2}{1 - \mathrm{R}_{\mathrm{i}}^2}\right) \tag{15}$$

This transformed variable Ψ_i converts R_i^2 from a bounded variable into an unbounded one (with a range from $-\infty$ to $+\infty$). Further, Ψ_i is highly correlated with R_i^2 but less skewed and less leptokurtic. As a consequence, the higher the value of Ψ_i, the more conformity there is to a particular investment style or multiple styles.

Table 2: **Regressions of Style on Pension Target Surplus Return**

$$\Psi_i = a_0 + a_1 SR_i + a_2 SR_i^2 + u_i$$

	Linear		Log z		Quadratic	
	(1)	(2)	(3)	(4)	(5)	(6)
	$z1$	$z2$	log($z1$)	log($z2$)	$z1$	$z2$
Constant	-2.08	-1.04	-3.00	-1.62	-14.66	-5.53
	(1.88)	(0.57)	(2.59)	(0.79)	(21.78)	(2.32)
Target Surplus Return	2.30	1.25	4.63	2.64	27.57	10.43
	(1.89)	(0.56)	(3.74)	(1.13)	(43.52)	(4.61)
Target Surplus Return^2					-12.67	-4.63
					(21.75)	(2.29)
R^2	0.006	0.012	0.006	0.014	0.007	0.026
F	2.38	5.22	2.43	6.01	1.49	5.53

Heteroskedasticity-robust standard errors are in parentheses.

Table 3: **Mean estimates of Ψ_i across industries**

	Industry	Mean Ψ	Standard Deviation
1.	Manufacturing	0.12	1.31
2.	FIRE	-0.26	1.08
3.	Mining	0.09	1.09
4.	Transportation	0.65	1.38
5.	Services	-0.22	1.15
6.	Retail	-0.14	1.13
7.	Wholesale	-0.07	1.82

The OLS estimates of (13) are presented in Table 2, the mean estimates of Ψ_i for 7 different industry groups are presented in Table 3. These categories include: Manufacturing, Finance/Insurance/Real Estate (FIRE), Mining, Transportation, Basic Services, Retail and Wholesale. The grouping seems instructive. We generally expect pension fund liabilities to change over time in response to earnings growth, changing interest rates, and demographic factors. The implication is that in declining industries where the base of active workers is declining we would expect pension liabilities to grow at a much lower rate than in growth industries where the base of active workers in increasing. We would further expect to find different R_i^2 and Ψ_i in declining versus growth industries. These results are borne out in Table 3.

3.2 Measuring Target Surplus Returns

Target surplus returns are constructed as the difference of the pension fund portfolio's expected return at time t (Compustat variable $LTROR$) and the fund's liability growth rate. Ideally, we would like to use the fund's discount rate that is used to compute PBO as the fund's liability return. Ordinarily, most firms will choose a high discount rate, as a higher rate results in lower pension plan contributions. In addition, higher discount rates effectively reduce the termination payments to workers who get laid off, change jobs, or retire. Unfortunately, because data collected by Compustat are 10-K reports filed with the Securities and Exchange Commission (SEC) and these filings are entirely voluntary, not all firms in the final sample disclose their discount rate data. On the other hand, firms' expected pension asset return rates are more frequently recorded[7]. We construct two definitions of target surplus returns: (a) $z1$ is the difference between a fund's portfolio expected return and time-average liability growth, and (b) $z2$ is the difference between a fund's portfolio expected return and initial liability growth (i.e. liability growth in 1990).

3.3 Regressions of Style on Target Surplus Return

We can test the proposition that a high target surplus return causes pension fund managers to become more conformists to a particular investment style or multiple styles by estimating

$$\Psi_i = a_0 + a_1 SR_i + a_2 SR_i^2 + u_i \qquad (16)$$

where SR_i represents the target surplus return on pension fund portfolio i. The results of estimating (16) are presented in Table 2.

The first two columns of Table 2 represent a linear univariate regression of Ψ on the two different interpretations of SR. Style increases with target surplus return which says that asset allocation increases with prominence as a plan aims for a higher level of funding. However, our model in equation (9) says that SR is nonlinearly related to asset allocation; we would therefore expect a better fit when we consider including SR in a nonlinear fashion. Columns 3 through 6 of the above table serve to incorporate nonlinearity of z. The signs on the SR's remain the same but based on the R^2 alone, the quadratic specification provides a better fit with $z2$ as the definition for the

[7] In instances where the asset return rates are missing for some years, we used a simple straight-line extrapolation.

target surplus return . Since *a priori* we do not preclude either definition of SR, we shall use $z2$ as our measure of target surplus return henceforth. Column 6 says that as target surplus return increases, the importance of style (*vis-a-vis* that of selection) increases (10.43), but at a decreasing rate (-4.63). Analyzing further, the results indicate that the importance of style increases until it is maximized at a surplus return of 13% and then declines thereafter[8].

Next, we want to test whether conformity and initial funding ratio as implied by the structural model (10) affect pension investment style. To test if conformity is attributable to industry effects, we use the mean of SR across the industry groups to which each pension plans belongs. We denote this measure as $\overline{SR}_k$ where k denotes an industry group (by a 2-digit SIC). $\overline{SR}_k$ is computed as the time and cross-sectional average of both the LTROR and the liability growth within industry k. Again, we would expect a positive sign on $\overline{SR}_k$; the industry acts as a peer group for each pension plan and is a natural benchmark against which pension trustees' performance are measured. A higher level of target surplus return from the group requires more strategic direction in asset allocation.

The results of the regressions are shown in Table 4. We see that conformity is a substantive ingredient in explaining the style of a pension plan. Trustees' reputation lies not only in how they manage pension assets but also how they perform against trustees within the same industry. We therefore expect a positive association between individual plan's target surplus return and its peer-group surplus return. To see how each plan's target surplus return is affected by what a peer group ex-ante belief, we regress individual firm's SR_{ik} on $\overline{SR}_k$:

$$\begin{array}{rcccccc} \hat{SR}_{ik} & = & -0.19 & + & 1.21 & \overline{SR}_k \\ & & (0.62) & & (0.63) & \end{array}$$

$$R^2 = 0.0082, \quad F = 3.46, \quad n = 421$$

If firms with higher initial funding ratios are more risk tolerant relative to firms with low funding ratios, then Ψ_i should be inversely related to the

[8] One can solve for a surplus return of 13% by calculus.

Table 4: **Regressions of Style on Pension Target Surplus Return, Conformity and Pension Plan Attributes**

	(1)	(2)	(3)	(4)
Constant	-24.48	-24.40	-24.50	-21.00
	(6.75)	(6.72)	(6.76)	(6.44)
Target Surplus Return	9,83	9.79	10.75	7.44
	(4.71)	(4.72)	(4.59)	(5.19)
Target Surplus Return Squared	-4.39	-4.36	-4.84	-3.33
	(2.34)	(2.35)	(2.28)	(2.55)
Conformity $\overline{SR}_k$	19.42	19.41	20.05	19.63
	(6.54)	(6.53)	(6.60)	(6.14)
Initial Funding Ratio		-0.05	-1.79	-2.08
		(0.25)	(1.03)	(0.99)
Initial Funding Ratio Squared			0.70	0.82
			(0.39)	(0.37)
$Hivol \times \overline{SR}_k$				-0.84
				(0.14)
R^2	0.042	0.043	0.046	0.12
F	6.16	4.62	4.03	9.28

Heteroskedasticity-robust standard errors are in parentheses.

initial funding level. Column 2 of Table 3 shows the regression result of including initial funding ratio as an independent variable. We see that the initial funding ratio is neither statistically significant nor is it substantive as a coefficient. However, since (10) suggests that initial funding ratio scales the argument in a trustee's utility function nonlinearly, Column 2 is misspecified. This suggests that previous studies might have been a little too presumptuous when they conclude that initial funding ratio does not matter. As can be seen in Column 3, when initial funding ratio enters quadratically, it substantively determines investment style. The results suggest that when initial funding ratio is high, pension plans rely less on asset allocation but more on security selection at an increasing rate to achieve a given target surplus return.

The reputation model asserts that plan trustees' beliefs is important to asset allocation decisions. A plan trustee who views the investment environment as being uncertain will invest differently from a plan trustee with an opposing view, holding all other factors constant. Specifically, the model predicts that a trustee who views his environment as being uncertain, will be more risk-tolerant and herd less. Conversely, a trustee who views his environment as being " safe " will herd more and be more risk-averse. We use the volatility of each individual pension surplus return as the relevant measure of uncertainty. Plans with a surplus volatility above the 75th percentile will be grouped as the high-volatility group and will have a value of 1 for the indicator variable *Hivol*. To test for whether a distinction between a high- and low- surplus volatility matters where asset allocation is concerned, we run regressions that test for differences in the slope of the conformity effect. That is, we interact $\overline{SR}_k$ with $Hivol$[9]. The results are shown in Column (4) of Table 4. The model predicts a negative slope on $Hivol \times \overline{SR}_k$ as plan trustees facing a more uncertain environment will tend to deviate more from its peers and conform less. Consequently, these trustees will rely more on

[9] We do not present the results of a $Hivol$-intercept specification here because the values of z fluctuates around 1 which in turn makes the indicator variable and the interaction term highly correlated. Consequently, the results of a model with an indicator intercept term and those from a model with a interaction-slope effect are very similar. Moreover, the model with the interaction term gives a better fit.

security selection than asset allocation. The results in Column(4) confirm this.

3.4 Firm Fixed Effects

In this section, we re-run our regressions allowing for firm fixed effects which are permanent unobserved factors that could potentially affect pension investment and are correlated with reported target surplus returns and funding ratios. For example, a firm's funding and investment philosophies and managerial style potentially affect return targets and funding status. For the fixed-effects model, we compute six 10-year rolling surplus returns for each firm and re-estimate the model. The results are shown in Table 5. Our results show that the relationship of pension investment style remains robust to the inclusion of fixed effects: the signs of the coefficients remain the same. All variables are statistically significant at the 1 percent level.

However, one has to be careful about the manner in which initial funding ratio is included in a fixed-effects estimation. In the current context, a firm's initial funding ratio (regressor) in one period could affect future period's investment style (dependent variable). Consequently, the unobservable errors u_{it} are likely to be correlated over time. Such time-dependence violates the *strict exogeneity* assumption required for the fixed-effects estimation. To account for this endogeneity issue, we allow initial funding ratios to be *sequentially exogenous* (instead of imposing strict exogeneity) to investment style. Practically, it means running a pooled OLS regression on first differences of the variables and substituting lagged initial funding ratios as instruments[10]. The results are shown in Table 6. The estimates confirm the results in the earlier regressions.

One could ascertain whether a fixed effects of a first-difference estimation is really necessary by performing a simple test for autocorrelation. We do this by regressing the residuals from the differenced equation on lagged values. A strong positive correlation in the levels implies a negligible correlation in the

[10]For details on sequential exogeneity, see Papke (1994).

Table 5: **Regressions of Style on Pension Target Surplus Return with Firm Fixed Effects**

$$\Psi_i = a_{0,it} + a_{1,it} SR_{it} + a_2 SR_{it}^2 + u_{it}$$

Observations = 421

	(1)	(2)	(3)
Target			
Surplus Return	0.50	2.44	2.52
	(0.12)	(0.73)	(0.73)
Target			
Surplus Return^2		-1.02	-1.16
		(0.38)	(0.38)
Conformity $\overline{SR}_k$			0.69
			(0.26)
R^2	0.77	0.77	0.77
F	16.32	16.25	16.33

Heteroskedasticity-robust standard errors are in parentheses.

differenced equation. If, however, there was no autocorrelation to begin with, then differencing would induce negative correlation. The AR1 coefficients for both regressions are -0.11 with standard errors of 0.02. These coefficients are statistically different from 0 which is the expected value if we are expecting a random walk. Moreover, the estimates are also different from -0.5 which is expected if there were no correlation in the errors in the levels. The results therefore indicate that neither a fixed effects nor a first-difference estimation is called for. The estimates in Table 6 are conservative since with induced serial correlation, OLS standard errors are likely to be overestimated.

4 Forming a Diversified Pension Fund Portfolio

In this section, we simulate our portfolio choice model given in (9), showing what asset allocation shares it predicts for real estate for the private pension plans in our sample. The return indices used to simulate (9) consist of the holding-period yields of five categories of assets including real estate, common stock, corporate bonds, 10-year government bonds, and treasury bills. The common-stock returns have two subcategories: large-stock returns and small-stock returns. The stocks and bonds returns data are obtained from the Ibbotson Associates' SBBI 2005 Yearbook, while the real estate returns series is taken from the NAREIT website.[11]

The initial funding ratio for each pension plan is calculated by dividing its pension assets by its PBO in 1990. A plan is overfunded if this ratio is greater or equal to 1 and is underfunded otherwise. There are 129 (31%) underfunded plans and 292 (69%) overfunded plans. The summary statistics of all plans categorized by their level of fundedness is presented in Table 7. The figures represent means and those in parentheses denote standard deviations. Overfunded plans in general have greater asset values and smaller

[11]A similar optimization exercise was performed using the NCREIF NPI total return series for real estate did not yield significant changes in the results. This is because (a) the two series have similar means and (b) their sample correlation within in the sample period is 0.99.

Table 6: **Regressions of Style on Pension Target Surplus Return**

$$\Delta\Psi_i = \delta_0 + \delta_1 \Delta SR_{it} + \delta_2 \Delta SR_{it}^2 + \delta_3 \Delta \overline{SR}_{k,t} + \delta_4 \Delta IF_{it} + \delta_5 \Delta IF_{it}^2 + \Delta u_{it}$$

Number of Observations = 2105

	(1)	(2)
Target Surplus Return	2.59	2.56
	(0.55)	(0.55)
Target Surplus Return^2	-1.31	-1.29
		(0.28))
Conformity $\overline{SR}_{k,t}$	0.68	0.68
	(0.16)	(0.16)
Initial Funding Ratio	-0.09	-0.55
	(0.07)	(0.32)
Initial Funding Ratio^2		0.18
		(0.12)
AR1	-0.11	-0.11
	(0.02)	(0.02)
R^2	0.023	0.024
F	12.37	10.34

Heteroskedasticity-robust standard errors are in parentheses.

R^2 and number of observations are from the first-differenced equation.

liabilities than do underfunded plans, their liability growth rate is slightly lower but are more volatile[12]. Except for the PBO and liability growth rate, there is relatively more variation among the overfunded plans than among the overfunded ones. Table 8 presents the unconditional summary statistics for these data over the sample period.

Table 7: Summary statistics of plans by their level of funding

	Underfunded Plans		Overfunded Plans	
No. of plans	129		292	
Pension asset ($ mil)	828	(3437)	967	(3607)
PBO ($ mil)	931	(4044)	829	(3096)
Initial funding ratio	0.89	(0.10)	1.23	(0.21)
Liability growth(%)	9.78	(4.64)	9.28	(4.11)
Liability volatility(%)	14.40	(4.88)	14.83	(5.37)

Table 8: Summary Statistics of Asset Returns

Asset	Mean Return (%)	Standard Deviation (%)	Skewness
Large stocks	12.42	18.42	-0.39
Small stocks	16.86	22.16	0.04
Corporate bonds	9.48	8.94	-0.22
Government bonds	9.84	11.30	0.00
Treasury bills	4.19	1.90	-0.21
Real estate	14.20	18.42	-0.52

Notice that on the basis of standard deviation alone, large stocks, small stocks, real estate and long-term US government bonds are considered rather risky, whereas US treasury bills and long-term corporate bonds are the ‘safer’ assets. However, one should bear in mind that the allocation optimization, by virtue of its entropic derivation as described in equation (9) considers

[12]Liability volatility is the standard deviation of a plan’s pension liability growth rate over the 15-year period.

beyond the first two central moments - namely, the mean and the variance - rather, the entire distribution is taken into account. Indeed, the entropic model's power utility form has the convenient property of its third derivative being positive as long as θ is positive. As Kane demonstrates, this embeds a preference for positively skewed returns. We provide the sample skewness for each of the asset class: small stocks are the most positively skewed, followed by government bonds and bills, with real estate being the most negatively-skewed asset class.

Table 9 summarizes the correlation matrix among the assets and firms' liabilities. The sample cross-correlations of the five asset classes' returns, within the sample period, range from -0.34 to 0.95 suggesting that a diversified portfolio across the assets will provide investor benefits. Movement of small stocks, in particular, seems to mirror that of real estate. This is interesting considering that both asset classes have rather high means and standard deviations, with the implication that the two asset classes could be substitutes in a portfolio. Pension liability is negatively related to the U.S. treasury bills, but this correlation is rather small. On the other hand, the movement of pension liability follows very closely to those of long-term corporate bonds and long-term government bond. Stocks and real estate are moderately correlated with pension liability.

Table 9: Correlation of asset returns 1990-2004 with pension liabilities.

	Large stocks	Small stocks	Corp. bonds	Govt bonds	T-bills	Real estate	Pension Liabilities
Large stocks	1.	0.62	0.13	0.12	0.15	0.24	0.15
Small stocks	0.62	1.	0.09	-0.04	-0.32	0.66	0.27
Corp. bonds	0.13	0.09	1.	0.95	0.09	0.27	0.81
Govt bonds	0.12	-0.04	0.95	1.	0.16	0.20	0.69
T-bills	0.15	-0.32	0.09	0.16	1.	-0.34	-0.25
Real estate	0.24	0.66	0.27	0.20	-0.34	1.	0.26
Liabilities	0.15	0.27	0.81	0.69	-0.25	0.26	1.

Table 10 contrasts the estimated allocations derived from the entropic model to a Sharpe ratio maximizing tangency portfolio (SRMP). Three levels of benchmark/target surplus returns, (0%, 2% and 5% respectively) are used to generate the entropic results. The holding period is assumed to be 15-years. The SRMP serves as an interesting comparison because it is based on a somewhat similar notion: the SRMP is the portfolio, among the portfolios along the mean-variance efficient frontier, with the highest probability of exceeding the risk-free rate. In this case, the risk-free rate acts as a floor return and is therefore an implicit benchmark. The Sharpe ratio is based on a surplus return (i.e. within the asset-liability framework) not unlike those used in previous studies (Peskin (1997) and Chun, Ciochetti and Shilling (2000)). For comparison, we assume that the constant riskless rate of interest, r_f, to be 0%, 2%, and 5% i.e.

$$SR_t = \frac{\left[\Sigma_j^6 \alpha_j E(R_{j,t}) - E(l_t)\right] - r_f}{\sigma_{\boldsymbol{\alpha}}} \tag{17}$$

where $E(l_t)$ is the cross-sectional average pension liability growth rate and $\sigma_{\boldsymbol{\alpha}} = Var(\Sigma_{j,t}\alpha_j R_{j,t} - l_t)$ is the variance of the surplus return.

The results in Table 10 are interesting. The SRMP predicts an average allocation of 52% to stocks, 36% to bonds and bills and 12% to real estate. One should bear in mind that the SRMP takes into account only total risk (via $\sigma_{\boldsymbol{\alpha}}$) and therefore does not consider portfolio diversification in its formation. Seen in this light, the near-uniform distribution of allocation among small stocks, bonds, bills and real estate is perhaps not too surprising.

The results in Table 10 also suggest that, as plans set higher target rates, the beating-the-benchmark attitude transforms into higher mean portfolio returns, accompanied by higher levels of standard deviations. Each of the entropic model has a lower portfolio variance than their respective SRMP counterpart. This is a direct consequence of the endogenous risk aversion parameter which is solved for in the entropic model but not in the SRMP framework. The SRMP does not consider risk aversion and only considers variance from the market and a plan's liability. We note that the risk aversion parameter does not have a monotonic relationship with the target return. Instead, as plans set more aggressive goals, there is a marked tilt toward stocks on average for both models. However, there is a difference between how the two models achieve higher portfolio returns. With higher z's, there

Table 10: Optimal portfolios of SRMP and entropic model for different levels of target surplus returns z (%)

	$z = 0\%$		z=2%		z=5%	
Asset Allocation	SRMP	Entropic	SRMP	Entropic	SRMP	Entropic
Large stocks	35.82	13.27	31.07	15.73	33.08	19.88
Small stocks	0.30	12.57	16.03	22.51	39.83	39.53
Corp. bonds	20.19	22.28	16.23	19.14	3.87	12.37
Gov. bonds	17.39	21.80	11.78	19.30	1.14	13.56
T-Bills	24.29	29.02	13.63	29.02	0.06	0.00
Real estate	2.01	1.07	11.27	6.28	22.03	14.67
Portfolio return	9.43	9.39	11.43	11.07	14.43	13.72
Portfolio S.D.	8.22	6.81	10.24	9.03	16.12	13.66
Risk aversion $1 + \theta^*_{\max}$	.	2.21	.	3.31	.	3.01

is an inclination for the entropic model to invest more heavily in small stocks. Such an outcome can be explained by the inherent skewness-preference that is embedded in the model: the third derivative of the entropic model is positive as long as concavity is imposed (i.e., $\theta > 0$). To achieve greater portfolio return, the entropic model predicts a higher investment in small stocks which is positively-skewed than real estate even though the two asset classes are similar in their means and standard deviations. The skewness-preference behavior is starker when one looks at the case where $z = 5\%$. Here, SRMP predicts a 1% allocation to government bonds whereas the entropic model predicts 13.6% in government bonds which ranks second among the asset classes in terms of skewness.

Overall, the results from the entropic model are heartening, we see the plausibility of having a crude 60-40 (bond-stocks) pattern that is predominant in existing plans' investments. We note that real estate investment ranges from 1 to 15% for the entropic model, which is modest compared to the 2 - 22% range for the SRMP. Prima facie, the entropic model's prediction seems to better reflect actual real estate allocation among pension funds.

Figure 3 shows the distribution of the optimal allocations among the six asset classes. The top and bottom lines represent the 90th and the 10th percentiles respectively. The middle line represents the median. Real estate allocation is skewed to the right with around 10% of the plans investing in 17-22% in real estate. More than half the plans do not invest in any real estate.

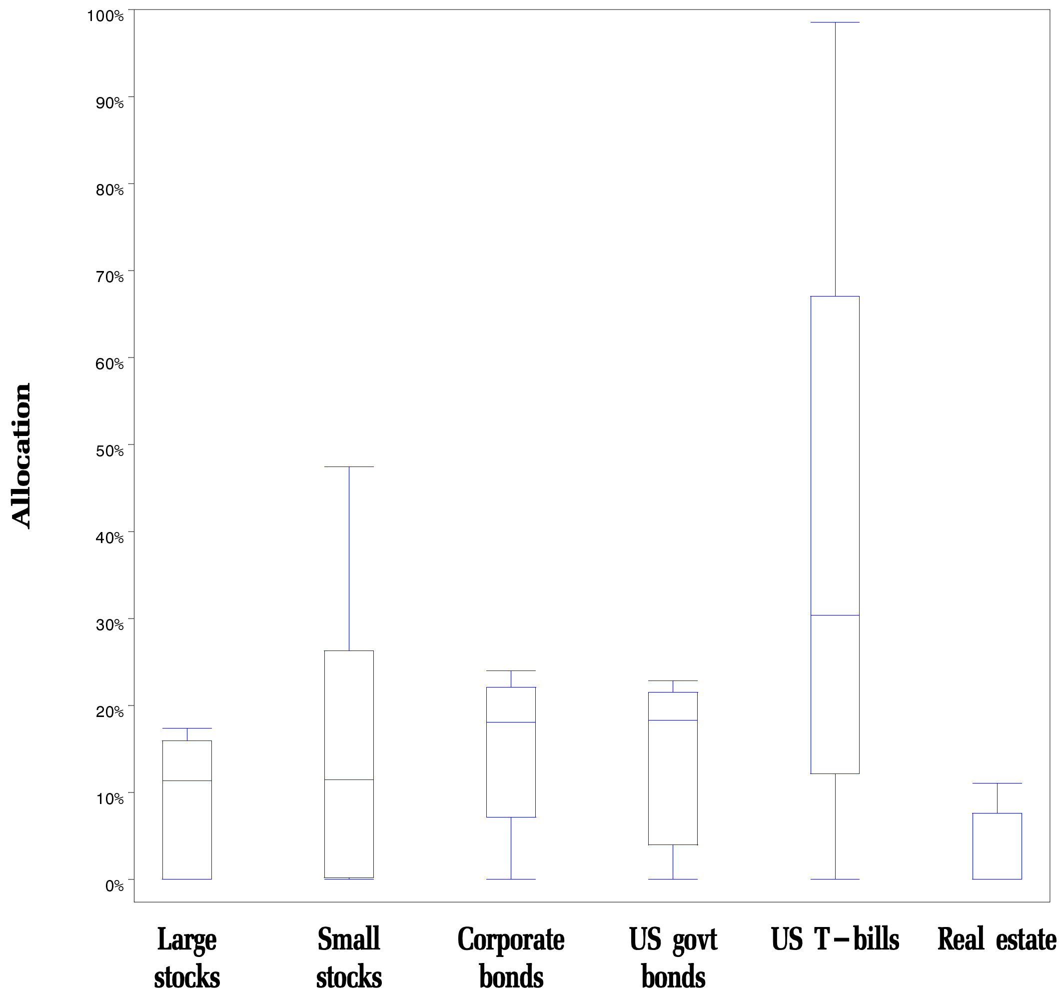

Figure 3: Distribution of optimal allocations

Pension plans' initial funding ratios can affect their allocation patterns. Presumably, plans with a high level of funding will have a greater level of cushion against market shocks than those plans with a lower level of funding. In this sense, plans with a higher funding ratio could potentially pursue assets that have higher rates of return. By this argument, an underfunded plan will not be able to pursue the same strategies as an overfunded fund. Since the endogenous risk aversion parameter is a function of both a plan's target as well as the investment opportunity set, we expect to see the risk aversion parameter vary with initial funding level. *A priori* the direction of this relationship is unclear. A plan with a high level of funding is well-placed to be more risk-loving, given the greater level of cushioning, but the causation could be argued in the other direction: a plan with a low level of risk tolerance could have a greater funding level due to its conservative investment style.

To demonstrate the importance of funding level on optimal allocation, we decompose our results by initial funding level quantiles for the three cases: a no-growth case (in Panel A), and target surplus returns at 2% (Panel B) and 5% (Panel C), respectively. The analyses are summarized in Table 11. Under the no-growth case, plans in the lowest funding ratio quantile are the most risk-loving and have larger allocations to both small stocks and real estate. Recall that in Tables 8 and 9, small stocks and real estate exhibit similar means and returns and have the highest correlation coefficient between each other. Across the different quantile groups, we see a tilting toward small stocks and real estate as targets increase. Moreover, the lowest 25-percentile group and the highest 25-percentile group exhibit the greatest risk-tolerance (i.e., lower risk aversion parameter values) and they are the ones who would derive higher payoff by having greater portfolio weight in real estate. It appears that real estate loses out to small stocks because of the positive-skewness preference behavior as explained above. Grouping the data by the initial funding ratio quantiles does not seem to give any strong pattern on the direction of the plans' risk aversion parameters and their respective fundedness except for the lowest and the highest 25-th quantile groups. The stocks-bonds-real estate allocation ratio in these two groups progresses from roughly 30-65-5 to 65-27-8 as target surplus return increases from 0% to 5%.

Table 11: Optimal allocation by initial funding ratio quantiles

Panel A
Target Surplus Return = 0%

Quantile	0 - 25%	26 - 50%	51 - 75%	75 - 100%
Large stocks	9.61	7.64	8.89	8.22
Small stocks	21.10	16.18	17.69	19.40
Corporate bonds	14.16	15.30	15.56	14.73
Government bonds	13.49	13.95	14.72	13.65
Treasury bills	36.78	43.55	40.00	40.58
Real estate	4.85	3.38	3.15	3.41
$\theta^*_{\max}$	1.21	2.23	1.75	2.18
Panel B Target Surplus Return = 2 %				
Large stocks	10.41	11.17	11.40	10.90
Small stocks	33.48	26.02	27.09	29.96
Corporate bonds	12.89	14.88	14.32	13.93
Government bonds	12.77	14.62	14.25	27.65
Treasury bills	23.86	27.67	26.86	25.59
Real estate	6.58	5.63	6.05	5.78
$\theta^*_{\max}$	1.82	1.88	1.82	1.76
Panel C Target Surplus Return = 5 %				
Large stocks	9.96	11.13	10.91	10.32
Small stocks	53.97	47.02	49.65	51.92
Corporate bonds	8.57	10.61	10.00	8.64
Government bonds	9.26	11.11	10.35	9.00
Treasury bills	10.04	10.82	11.46	12.00
Real estate	8.2	7.31	7.38	8.20
$\theta^*_{\max}$	1.55	1.52	1.55	1.7

4.1 Robustness Check

Direct real estate return series suffers from appraisal smoothing. It has often been argued that mean-variance analyses that use appraisal data overstate the desirability of real estate in pension funds' portfolios because the volatility of returns that are obtained from appraisal data are severely understated. The right approach, according to this argument, is to reverse-engineer the observed real estate appraisal series to create a hypothetical, desmoothed series that can be then fed into a mean-variance analysis. Table 12 shows that the results produced by the entropic model is robust to both the Fisher, Geltner and Webb (1987), FGW technique and the Cho, Shilling, Kawaguchi, CSK (2003) technique. The optimal allocation to real estate is slightly higher under the CSK technique but both desmoothed series produce predictions that are less than the prediction produced using the smoothed NCREIF NPI series. Pension plans are expected to decrease their allocation to real estate when targets are set higher. Again, there is a marked tilt to small stocks when a high portfolio yield is desired.

5 Conclusions

This paper has proposed a new way of thinking about the underinvestment problem in real estate assets by institutional investors, suggesting that the reason pension fund trustees will generally skew their holdings of assets away from real estate is to achieve a minimum required target rate of return. Failing to achieve this target return is associated with a loss of reputation. This way of thinking about pension fund trustee behavior proves to be quite appealing, in that it can help explain a heretofore puzzling stylized fact: why actual allocations to real estate by institutional investors are observed to be only $2\frac{1}{2}$-4% of total assets, whereas normative amounts range from 15-45% of total assets.

The paper also shows that this minimum required target rate of return will generally lead pension fund trustees to be conformists. This notion is quite consistent with Keynes' well-known beauty-contest argument, in which it is better for reputation to fail conventionally. We find support for the hypothe-

Table 12: Optimal Allocation using NCREIF NPI and different desmoothing techniques.

Using NCREIF NPI series			
	z=0%	z=2%	z=5%
Large stocks	9.09	11.64	12.92
Small stocks	16.73	29.64	50.72
Corporate bonds	14.31	14.03	11.00
Government bonds	13.74	13.80	11.50
Treasury bills	38.94	25.60	11.45
Real estate	7.19	5.29	2.41
$\theta^*_{\max}$	2.78	2.77	2.72
Desmoothed real estate series using the FGW technique			
Large stocks	9.22	11.30	12.81
Small stocks	18.04	28.40	51.15
Corporate bonds	14.66	14.90	10.80
Government bonds	13.70	14.40	11.24
Treasury bills	41.30	27.80	11.60
Real estate	3.14	3.31	2.42
$1 + \theta^*_{\max}$	2.68	2.85	2.64
Desmoothed real estate series using the CSK technique			
Large stocks	10.00	11.40	11.21
Small stocks	19.90	29.90	52.40
Corporate bonds	14.20	14.10	10.70
Government bonds	13.60	13.68	11.17
Treasury bills	38.60	26.49	10.70
Real estate	5.50	4.46	1.84
$1 + \theta^*_{\max}$	2.74	2.85	2.54

sis that a high target surplus return indeed causes pension fund managers to become more conformists to a particular investment style or multiple styles. The results also suggest that conformity effects are weaker when volatility increases.

We should emphasize that we do not view achieving a minimum required target rate of return as the sole explanation for why institutional investors devote very little resources to investing directly in real estate. Rather, we see our explanation as complementary to others, emphasizing, for instance, that investors hold assets that they know of more, or that institutional investors are affected by prudence restrictions to varying degrees.

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